JOURNAL OF PROPULSION AND POWER
Vol. 10, No. 4, July–Aug. 1994

Improved Method for Estimation of the Maximum Instantaneous Distortion Values

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In this article a statistical method for estimating the maximum instantaneous distortion values is developed further to estimate the maximum values of the instantaneous distortions in which the operation of peak values of dynamic pressures and correlation between them are involved. The effects of the correlation between fluctuating pressures on the estimation are discussed. Comparison between the estimated values and the experimental results shows that the developed method can quite accurately estimate the maximum value of the instantaneous distortion with operation of peak values of dynamic pressures involved, such as the distortion descriptor DC_θ. The accuracy of estimation of maximum instantaneous distortion can generally be improved by considering the correlations between fluctuation pressures. However, when the mean correlation coefficient $\bar{\rho}$ is less than 0.1, the estimation can be effected by dispensing with correlation.

Nomenclature

A, B, C, D, E = coefficients defined in calculation of distortion
$DC_{K\theta}$ = defined distortion coefficient in a given θ-deg sector
DC_θ = distortion coefficient based on θ-deg sector
$E[\,]$ = mean value operator
fc = cutoff frequency of a low pass filter
fs = sampling frequency
I = number of rings of pitot reading
J = number of pitot readings per ring
K = distortion coefficient
K_{A2}, K_{RAD}, K_θ = distortion coefficients
M = number of pitot readings per ring in a θ-deg sector
P = dynamic total pressure
P_{ss} = steady-state total pressure
P_{ssav} = mean value of steady-state total pressure over the whole face considered
P_t = fluctuation pressure
q = dynamic pressure head, $\frac{1}{2}\rho V^2$
T = time duration
t = time
ρ = correlation coefficient between fluctuation pressures
σ_k = the rms value of instantaneous distortion

Subscripts
av = mean value over the whole face
CO = values estimated by considering correlation between fluctuation pressures
CR = values estimated by considering simultaneously the correlation between fluctuation pressures and the actual distribution of the amplitude probability density of the instantaneous distortion
i, j, k, l = coordinate of the measured points
M = experimental values
max = maximum values
rms = root-mean-square values
ss = steady-state values
UN = values estimated with assumptions of uncorrelated fluctuation pressures and Gaussian instantaneous distortion
UR = values estimated by considering the actual distribution of amplitude probability density of instantaneous distortion and with assumption of uncorrelated fluctuation pressures

Superscript
$-$ = time average or mean values

Introduction

TO lower the cost of finding the maximum instantaneous distortion values by deterministic method, three well-known statistical methods[1] for estimating the maximum instantaneous distortion value have been developed in the U.S. since 1972. Another new statistical method for the estimation is described in Ref. 2. By the latter, the time average and rms values of instantaneous distortion descriptors are obtained analytically from the measured steady-state inlet total pressures and rms values of fluctuation pressures which are considered to be uncorrelated with each other. In turn, the maximum values are predicted by assuming that the distribution of the amplitude probability density (APD) of the instantaneous distortion is Gaussian. In Ref. 3, it is revealed that the APD of the instantaneous distortion is not always Gaussian and is dependent on the steady-state distortion values and rms values of fluctuation pressures. The method for estimating maximum value of instantaneous distortion in Ref. 2 is further improved as in Ref. 3 by considering the actual distribution of the APD of the instantaneous distortions, and the accuracy of estimation is improved.

The apparent advantage of the method stated in Ref. 2 is that only 3 s of computing time on an Intel 86/330 microcomputer is required to estimate the maximum instantaneous distortion value for 40 probes' data. Unfortunately, this method can only be used to estimate the maximum value of instantaneous distortion indices K_θ, K_{rad}, K_{A2},[4] as in the calculation of the distortion only algebraical operations of the dynamic pressures are involved, whereas for an index such as DC_θ,[4] operations of peak values of the dynamic pressures are involved and the method of Ref. 2 appears to be futile. To

Received June 15, 1992; revision received Sept. 15, 1993; accepted for publication Jan. 4, 1994.
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make the method of Ref. 2 more useful, it is necessary to extend it further.

In the estimation of maximum instantaneous distortion values by the method in Ref. 2, uncorrelated fluctuation pressures are assumed. Nevertheless, as the correlation between fluctuation pressures at the outlet of the real inlets does exit, it can never be neglected in some cases. It would be expected that the accuracy of the estimation will be improved by considering the correlation between fluctuation pressures in the estimation of maximum values.

In this article, the method described in Ref. 2 is further extended to estimate the maximum values of the instantaneous distortion with operations of peak value involved and those with correlation between fluctuation pressures considered, respectively. Moreover, the effects of correlation between fluctuation pressures on the estimation of maximum instantaneous distortion are discussed.

Estimation of Maximum Value of Instantaneous Distortion with Operations of Peak Value Involved

Some Considerations About the Operation of Peak Value in Distortion Index DC_θ

According to definition, the distortion DC_θ can be written as

$$DC_\theta = (P_{\min\theta} - P_{av})/q_{av} \tag{1}$$

where P_{av} is the mean value of the dynamic pressure over the whole face at time t

$$P_{av} = \frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} P_{ij} \tag{2}$$

$P_{\min\theta}$, the minimum value of the mean total dynamic pressures taken in sectors of angle θ on the face, may be expressed as

$$P_{\min\theta} = \min_K \left(\frac{1}{IM}\sum_{i=1}^{I}\sum_{j=K}^{K+M} P_{ij}\right), \quad k = 1, \ldots, J-1 \tag{3}$$

submitting Eqs. (2) and (3) into Eq. (1), we can get

$$DC_\theta = -\max_K \left[\frac{1}{q_{av}}\left(\frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} P_{ij} - \frac{1}{IM}\sum_{i=1}^{I}\sum_{j=K}^{K+M} P_{ij}\right)\right]$$

Based on the idea of maximum instantaneous distortion value within a given time interval, the maximum value of instantaneous distortion $DC_{\theta\max}$ can be expressed as

$$DC_{\theta\max} = -\max_t \max_K (DC_{K\theta}) \tag{4}$$

where

$$DC_{K\theta} = \frac{1}{q_{av}}\left(\frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} P_{ij} - \frac{1}{IM}\sum_{i=1}^{I}\sum_{j=K}^{K+M} P_{ij}\right) \tag{5}$$

Equation (5) gives a distortion descriptor, defined in this article, in a given sector of angle θ, similar to distortion descriptor DC_θ. It can be seen from Eq. (5) that there is no operation of peak value of dynamic pressures involved in this distortion descriptor. From Eq. (4), the maximum value of the instantaneous distortion DC_θ may be obtained by finding the peak value of $DC_{K\theta}$ among the sectors at a given time, then determining the maximum value of the peak value in a given duration. But it is very difficult to find the maximum value of the instantaneous distortion in this way. Fortunately, according to the rule of operation of the peak values, Eq. (4) can be written into

$$DC_{\theta\max} = -\max_K \max_t DC_{K\theta} = -\max_K DC_{K\theta\max} \tag{6}$$

Equation (6) shows that the maximum value of the instantaneous distortion DC_θ can be obtained by another way, i.e., to estimate the maximum value of instantaneous distortion $DC_{K\theta}$ of a given sector in the given duration by the method described in Ref. 2 beforehand, and then to determine the peak value of the maximum values $DC_{K\theta\max}$ of the sectors over the whole face.

Maximum Value of Instantaneous Distortion

There is no operation of peak value of dynamic pressure in the distortion $DC_{K\theta}$ as defined in Eq. (5). Thus, the maximum instantaneous distortion value $DC_{K\theta\max}$ can be estimated by the method in Ref. 2. That is, the time average and rms values of the instantaneous distortion can be analytically obtained through the magnitude-order analysis from the measured total steady-state and rms values of dynamic pressures with an assumption of uncorrelated fluctuation pressures. Then the maximum instantaneous distortion value is estimated with the most probable probability at which the maximum instantaneous distortion value occurs in duration T and the distribution of APD of instantaneous distortion.

The estimated and experimental maximum values of the instantaneous distortion descriptor DC_{90} within 45 s, are given in Fig. 1. The experimental deterministic results, given in Ref.

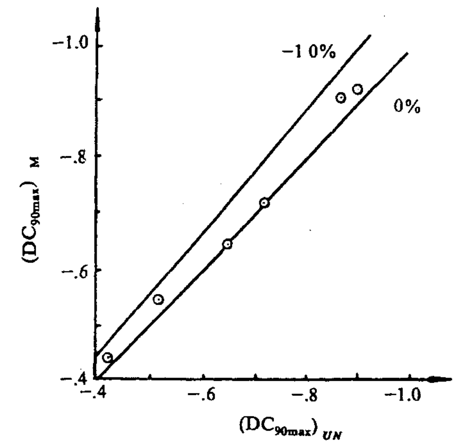

Fig. 1 Comparison between the experimental and estimated maximum instantaneous distortion values with an assumption of Gaussian instantaneous distortion.

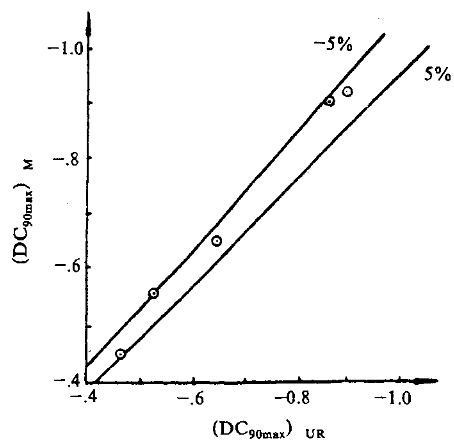

Fig. 2 Comparison between the experimental and estimated maximum values of instantaneous distortion by considering the actual distribution of APD of instantaneous distortion.

2, at flow speed of Mach number 0.3–0.5, and the maximum average turbulence of 2.2%, are denoted by subscript M. The estimated values are obtained by the developed method given above with assumptions of uncorrelated fluctuation pressures and Gaussian instantaneous distortion. It can be seen from the figure that the method described in this article for distortion DC_θ with operations of peak value can quite accurately estimate the maximum instantaneous distortion values. Comparison between the test results and estimated values predicted by the method by considering the actual distribution of APD of instantaneous distortion[3] is shown in Fig. 2. In this case, the error of estimation of maximum value is reduced to 5%. It is clearly shown in comparing Figs. 1 and 2 that the accuracy of estimation of the maximum value of instantaneous distortion with peak operations of dynamic pressures can be improved by considering the actual distribution of the APD of the instantaneous distortion.

Estimation of Maximum Value of Instantaneous Distortion by Considering Correlation Between Fluctuation Pressures

Time Average, rms, and Maximum Values of the Instantaneous Distortion

As discussed in Ref. 2, the instantaneous distortion descriptors K_θ, K_{rad}, K_{A2} and $DC_{K\theta}$, as defined earlier in this article, can be represented as

$$K = \sum_{i=1}^{I}\sum_{j=1}^{J}\left(A_{ij} + B_{ij}\frac{P_{tij}}{P_{\text{ssav}}} + C_{ij}\frac{P_{tij}^2}{P_{\text{ssav}}^2}\right) + \sum_{i=1}^{I}\sum_{\substack{j=1 \\ (i\neq k)\cap(j\neq l)}}^{J}\sum_{k=1}^{I}\sum_{l=1}^{J} D_{ij}\cdot E_{kl}\frac{P_{tij}\cdot P_{tkl}}{P_{\text{ssav}}^2} +, \ldots \tag{7}$$

As Eq. (7) gives the steady-state distortion when the fluctuation pressures are zero, i.e., $P_t \equiv 0$, it can be rewritten into

$$K = K_{\text{ss}} + \sum_{i=1}^{I}\sum_{j=1}^{J}\left(B_{ij} + \frac{P_{tij}}{P_{\text{ssav}}} + C_{ij}\frac{P_{tij}^2}{P_{\text{ssav}}^2}\right) + \sum_{i=1}^{I}\sum_{\substack{j=1 \\ (i\neq k)\cap(j\neq l)}}^{J}\sum_{k=1}^{I}\sum_{l=1}^{J} D_{ij}\cdot E_{kl}\frac{P_{tij}\cdot P_{til}}{P_{\text{ssav}}^2} +, \ldots \tag{8}$$

Let ρ_{ijkl} represent the correlation coefficient between fluctuation pressures between the measured points (i, j) and (k, l), i.e.

$$\rho_{ijkj} = \frac{E(P_{tij}\cdot P_{tkl})}{P_{rmsij}\cdot P_{rmskl}} \tag{9}$$

Similar to the way of magnitude-order analysis discussed in Ref. 2, by considering the correlation between fluctuation pressures, the time average and rms values of instantaneous distortion are as follows:

$$\bar{K} = K_{\text{ss}} + \sum_{i=1}^{I}\sum_{j=1}^{J} C_{ij}\frac{P_{rmsij}^2}{P_{\text{ssav}}^2} + \sum_{i=1}^{I}\sum_{\substack{j=1 \\ (i\neq k)\cap(j\neq l)}}^{J}\sum_{k=1}^{I}\sum_{l=1}^{J} D_{ij}\cdot E_{kl}\cdot\rho_{ijkl}\frac{P_{rmsij}\cdot P_{rmskl}}{P_{\text{ssav}}^2} \tag{10}$$

$$\sigma_k^2 = \sum_{i=1}^{I}\sum_{j=1}^{J} B_{ij}^2\frac{P_{rmsij}^2}{P_{\text{ssav}}^2} + \sum_{i=1}^{I}\sum_{\substack{j=1 \\ (i\neq k)\cap(j\neq l)}}^{J}\sum_{k=1}^{I}\sum_{l=1}^{J} B_{ij}\cdot B_{kl}\cdot\rho_{ijkl}\frac{P_{rmsij}\cdot P_{rmskl}}{P_{\text{ssav}}^2} \tag{11}$$

The maximum instantaneous distortion values are respectively estimated in the same way as that in Refs. 2 and 3, i.e., by Eq. (11) in Ref. 2, or Eq. (12) in Ref. 3. Let $Z_{\max}$ represent the value of the standard probability function corresponding to the most probable probability at which the maximum instantaneous distortion occurs in duration T, the maximum value of instantaneous distortions can be expressed as

$$K_{\max} = \bar{K} + Z_{\max}\cdot\sigma_k \tag{12}$$

From Refs. 2 and 3, $Z_{\max}$ is determined by fs, the duration T for estimation, and the distribution of APD of the instantaneous distortion. Nevertheless, it is not related to the correlation between fluctuation pressures.

By the analysis of and comparison between Eqs. (10) and (11) in this article, and Eqs. (8) and (9) in Ref. 2, it is not very difficult to find that the last terms in Eqs. (10) and (11) are added for considering the correlation between fluctuation pressures. This is why the accuracy of estimation of time average, rms, and maximum values of instantaneous distortion can be improved after considering the correlation between fluctuating pressures shown in this article. The time average value of instantaneous distortion is not equal to the steady-state distortion value, yet is generally greater.

Effects of the Correlation Between Fluctuation Pressures on the Accuracy of Estimation

It can be seen that only the value of elements $(\bar{K} - K_{\text{ss}})$, $(K_{\max} - K_{\text{ss}})$, and σ_k are affected by the fluctuation pressures, and so these three elements will be used to discuss the effects of the correlation between fluctuation pressures on the accuracy of estimation. The relative error of the estimated maximum value given in Ref. 2, with assumptions of not-corre-

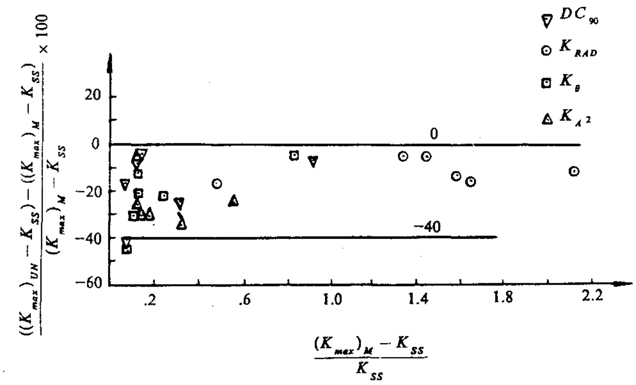

Fig. 3 Relative error of maximum instantaneous distortion value estimated by an assumption of no correlation between fluctuation pressures.

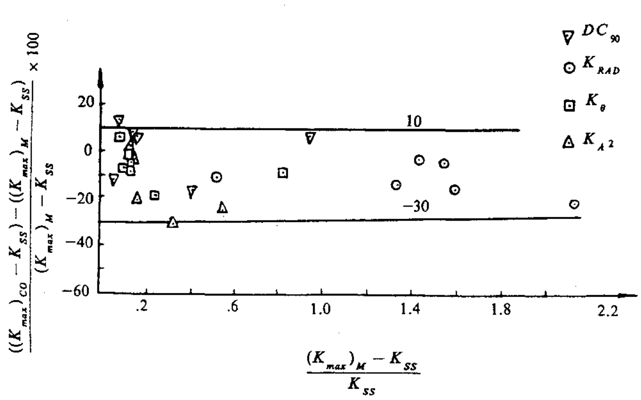

Fig. 4 Relative error of maximum instantaneous distortion value estimated by considering the correlation between fluctuation pressures.

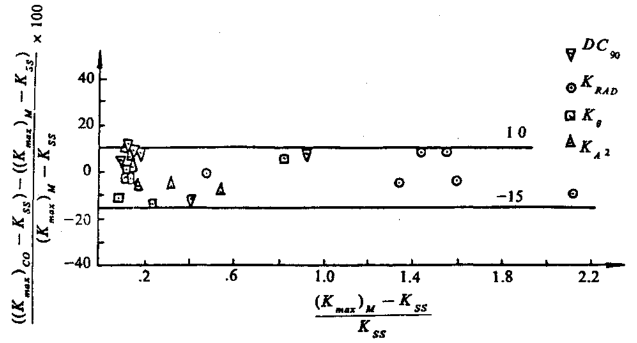

Fig. 5 Relative error of maximum instantaneous distortion value estimated by considering simultaneously correlation between fluctuation pressures and the actual distribution of the APD of the instantaneous distortion.

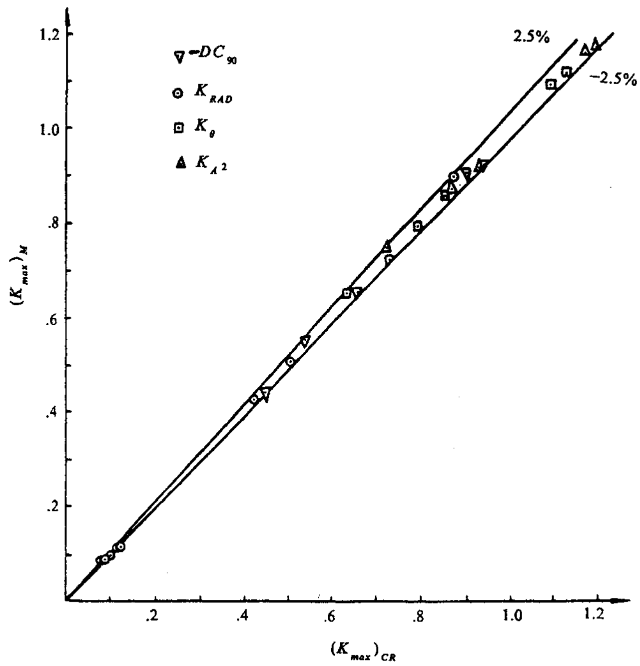

Fig. 6 Comparison between the experimental and estimated maximum values of instantaneous distortion by considering simultaneously correlation between fluctuation pressures and the actual distribution of the APD of the instantaneous distortion.

lated fluctuation pressures and a Gaussian instantaneous distortion based on the experimental results[2] is given in Fig. 3. The relative error, based on the same experimental data, of maximum value estimated by considering the correlation between fluctuation pressures [Eqs. (10) and (11)], and with an assumption of a Gaussian distribution of instantaneous distortion [Eq. (11) in Ref. 2], is given in Fig. 4. It can be seen from Figs. 3 and 4 that the relative errors of maximum value of instantaneous distortion can be reduced by considering the correlation between fluctuation pressures. The average relative error of

$$\frac{1}{N}\sum^{N}\left|\frac{(K_{\max} - K_{ss}) - [(K_{\max})_M - K_{ss}]}{(K_{\max})_M - K_{ss}}\right|$$

is reduced to 11.6% by considering the correlation between fluctuation pressure from 19.5% with the assumption of not-correlated fluctuation pressures, where N is the number of data in the figure. Similar to the discussion about $K_{\max}$ above, the average relative errors of element $(\bar{K} - K_{ss})$ and σ_k are respectively reduced to 12.33% from 28.16%, and to 5.43% from 11.37%, due to consideration of correlation between fluctuation pressure. It can be seen from the Figs. 3 and 4 that the accuracy of estimation of maximum values of instantaneous distortions can be improved by considering correlation between fluctuation pressures.

Figure 5 gives the relative error of the maximum values estimated by considering simultaneously the correlation between fluctuation pressures and the actual distribution of the APD of the instantaneous distortion [Eq. (12) in Ref. 3], the average relative error of maximum value is further reduced to 7.91%. Figure 6 gives the accuracy of estimation of the maximum value corresponding to Fig. 5. It is clear from the comparison between Fig. 6 in this article and Fig. 4 in Ref. 3, that the accuracy of estimation can be further improved with the actual distribution of APD of instantaneous distortion, even in the case of considering the correlation between fluctuation pressures.

Discussion

Estimated results and discussion show that the accuracy of estimation can be improved by considering the correlation between fluctuation pressures, even in the cases of low turbulence level and less correlations between fluctuation pressures. But it is more difficult and expensive to get the correlation coefficients between fluctuation pressures in practice. Thus, it is necessary to find whether or when the effects of correlation of pressures on the accuracy of estimation can be neglected. In Fig. 7, the relative errors (referred to values estimated with the assumption of no correlation between fluctuation pressures), of estimation of maximum instantaneous distortion values predicted by considering the correlations between fluctuation pressures with different turbulence levels and correlations between fluctuation pressures. In the figure, the horizontal axes represent the mean correlation coefficients between fluctuation pressures, defined as

$$\bar{\rho} = \frac{\dfrac{1}{IJ-1}\displaystyle\sum_{i=1}^{I}\sum_{j=1}^{J}\underset{(i\neq k)\cap(i\neq l)}{\sum_{k=1}^{I}\sum_{l=1}^{J}} \rho_{ijkl}P_{rmsij}\cdot P_{rmskl}}{\displaystyle\sum_{i=1}^{I}\sum_{j=1}^{J}P^2_{rmsij}} \tag{13}$$

and the total steady-state pressure distortions and turbulence of each configuration are real inlet experimental data with cutoff frequency, fc = 200 Hz, of the low-pass filter and sampling frequency fs = 1000 Hz. Of all the configurations, the steady-state distortions are quite different from each other, ranging from 0.2362 to 1.0203 for K_θ, 0.0272–0.3438 for K_{rad}, 0.5300–1.0475 for K_{A2} and 0.0084–0.0159 for the average turbulence. The correlation coefficients between fluctuation pressures are purposely contemplated to reflect the effect of correlation of pressures on the accuracy of estimation. It can be seen from these figures that the difference between maximum instantaneous distortion values estimated by considering the correlation and assuming no correlation between fluctuation pressures will increase almost linearly with the increase of the mean correlation coefficient $\bar{\rho}$, and when $\bar{\rho} \leq 0.1$

$$\left|\frac{(K_{\max})_{CO} - (K_{\max})_{UN}}{(K_{\max})_{UN}}\right| \leq 3\%$$

It is clear from Fig. 7 that the maximum instantaneous distortion value can be estimated by the assumption of no correlation of fluctuation pressures when $\bar{\rho} \leq 0.1$, with about the same accuracy of estimation as that by considering the correlation between fluctuation pressures.

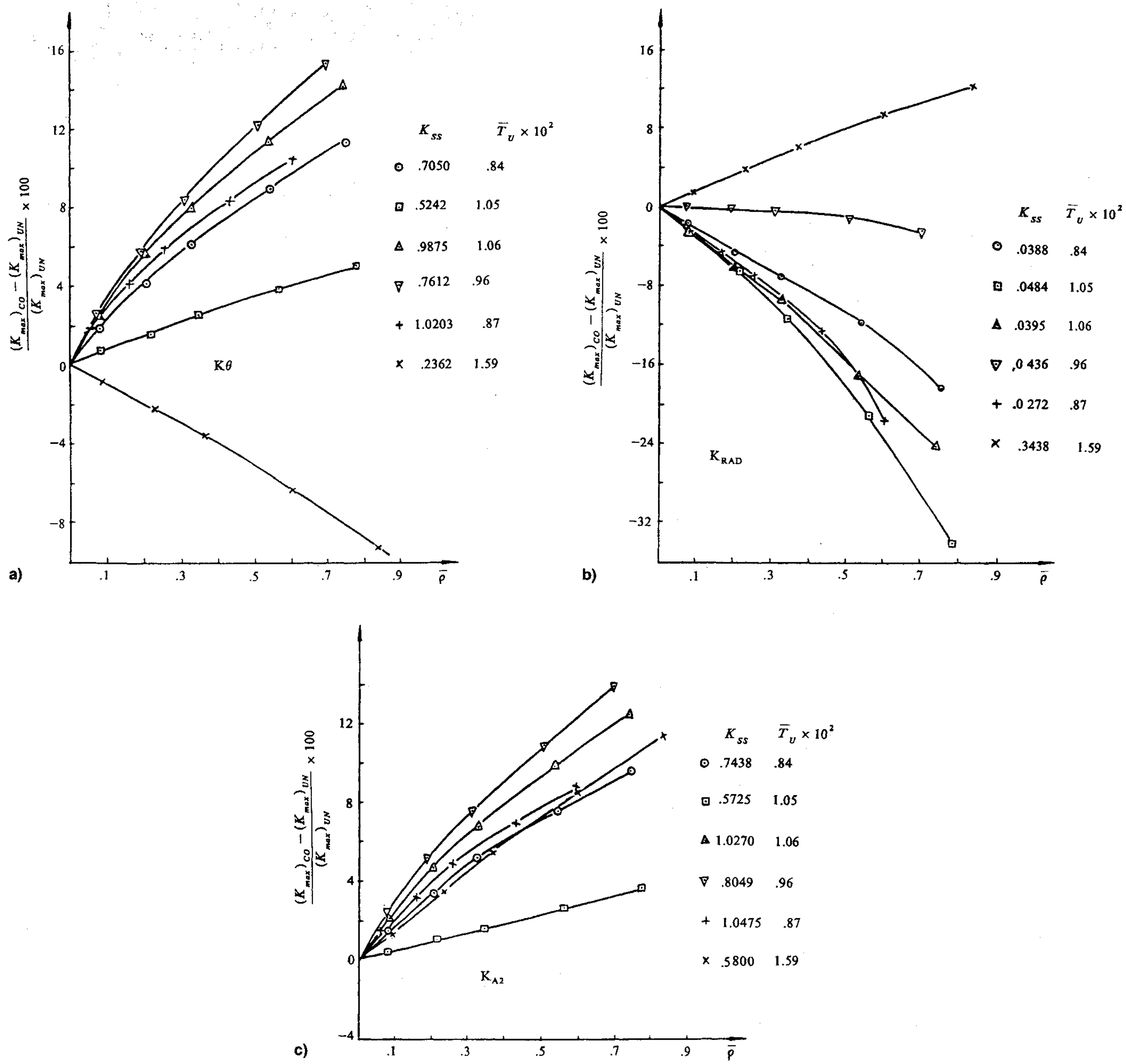

Fig. 7 The effect of mean correlation coefficient on the relative error of maximum instantaneous distortion values: a) K_θ, b) K_{rad}, and c) K_{A2}.

Conclusions

The following conclusions can be drawn from the present investigation:

1) The maximum value of instantaneous distortion with operations of peak value such as DC_θ can be estimated quite accurately by the method in Ref. 2 after some special treatment of the operation of distortion index, and the basic idea of the estimation of distortion with operations of peak value can be used for other similar distortion index as well.

2) The accuracy of estimation can be improved by considering the correlation between fluctuation pressures, and more accurate results may be obtained by considering simultaneously the correlation between fluctuation pressures and the actual distribution of the APD of the instantaneous distortions.

3) When the mean correlation coefficient $\bar{\rho}$ is less than 0.1, the maximum value of instantaneous distortion can be estimated by the assumption of uncorrelated fluctuation pressures instead of being estimated by considering the correlation between fluctuation pressures with negligibe change of accuracy.

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